Prop $\exists$ a long exact

$$\cdots \to H_n(A) \to H_n(X) \to H_n(X,A) \to H_{n-1}(A) \to \cdots$$

Cor $H_n(X, \emptyset) \cong H_n(X)$

Since $H_n(\emptyset) = 0 \ \forall n$.

Def A s.e.s splits

$$0 \to A \xrightarrow{i} B \xrightarrow{p} C \to 0 \quad \text{iff}$$

$\exists\ s : C \to B$ s.t. $s \circ p = id_C$

Ex. If C is free then the sequence always splits.

Ex $0 \to \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \to \mathbb{Z}_2 \to 0$ does not split.

Def Reduced Homology Define

$$\tilde{C}_n(X) = \begin{cases} C_n(X) & n \geq 0 \\ \mathbb{Z} & n = -1 \\ 0 & \text{else} \end{cases}$$

$C_0(X) \to \mathbb{Z}$

$\sum n_\sigma \sigma \mapsto \sum n_\sigma$.

$\tilde{H}_n(X) = H_n(\tilde{C}_*(X))$.

Note this only differs for dim. 0 for $X \neq \emptyset$ and in dim 0 we have a s.e.s

$\tilde{H}_0(X) \to H_0(X) \to \mathbb{Z}$

Since $\mathbb{Z}$ is free this splits and

$$H_0(X) \cong \tilde{H}_0(X) \oplus \mathbb{Z}$$

Def X is acyclic if $\tilde{H}_n(X) = 0 \ \forall n$.

Note $\tilde{H}_n(X,A) \cong H_n(X,A)$ if $A \neq \emptyset$