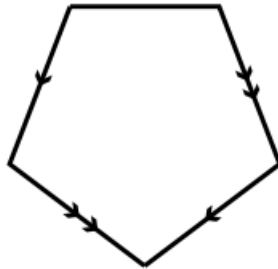


Problem Sheet 1

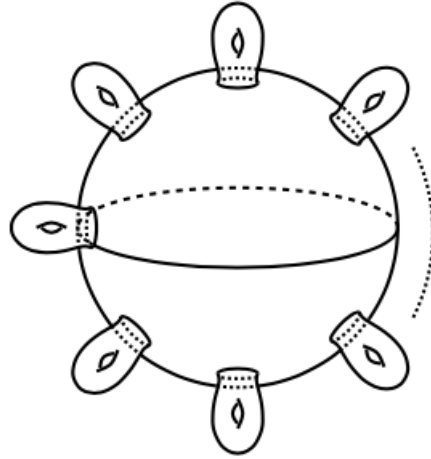
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1. Draw the Cayley graphs for
 - (a) $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ with generating set $S = \{(0, 1), (1, 0)\}$.
 - (b) $G = D_8$, the dihedral group of order 8 (symmetries of a square) with generating set $S = \{\sigma, \tau\}$, where σ is a rotation of order 4 and τ is a reflection.
 - (c) $G = D_8$, the dihedral group of order 8 (symmetries of a square) with generating set $S = \{\sigma, \tau\}$, where σ is a reflection through the midpoint of an edge and τ is a reflection through a vertex.
2. Show that the n -simplex is a simplicial complex. How many simplices are there in the n -simplex?
3. Let $K = (V, \Sigma)$. Show that there is a simplicial map to the $(|V| - 1)$ -simplex.
4. Prove that the triangulation $K = (\{1, 2, 3\}, \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\})$ of S^1 is minimal. Here minimal means that there is not a triangulation with fewer simplices.
Hint: Consider the possible subcomplexes of the n -simplex for $n < 2$.
5. Let K be a simplicial complex with n simplices. What is the maximal possible dimension of K .
6. (a) Show that a torus with a disk removed is homeomorphic to the image depicted below.



- (b) Deduce that the torus with a disk removed can be given a cell structure with 1 0-cell, 3 1-cells and 1 2-cell.
- (c) Let S be the surface of genus g depicted below.



Show that this can be constructed as follows. Start with a $4g$ -sided polygon, and identify its sides in pairs, according to the following recipe:



[Hint: divide the polygon up into g pentagons and a g -sided polygon.]

- (d) Deduce that the surface of genus g can be given a cell structure with 1 0-cell, $2g$ 1-cells and 1 2-cell.
7. (Optional) Give a triangulation of the cell complex below. This complex is the known as the dunce hat.



8. (Optional) Give a cell structure on the 3-torus $S^1 \times S^1 \times S^1$.

9. (Optional) Given $v \geq 1, e \geq 0$ and $f \geq 1$ with $e \geq v$ such that $v - e + f = 2$ find a cell structure on S^2 with v vertices, e edges and f 2-cells
10. (Doubly Optional) Remove the assumption $e \geq v$ from the previous question.