# Algebraic Topology Sheet 3

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1. Show that $\pi_1(X \times Y, (x, y)) = \pi_1(X, x) \times \pi_1(Y, y)$. (Hint: use the projections $X \times Y \to X$ and $X \times Y \to Y$.)

2. A *retraction* of $X$ onto $A \subset X$ is a map $r \colon X \to A$, such that $r \circ i \colon A \to A$ is the identity map.

   (a) Show that there is no retraction $D^2$ onto $S^1$.

   (b) Show that every map $f \colon D^2 \to D^2$ has a fixed point. To start you off, suppose that $f(x) \neq x$ for all $x \in D^2$. Use the pairs $(x, f(x))$ to construct a retraction $D^2$ onto $S^1$, reaching a contradiction. Thus, $f$ must have a fixed point.

3. Show the following:

   (a) $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^n$, $n \geq 2$.

   (b) $\mathbb{R}^2$ is not homeomorphic to $\mathbb{R}^n$, $n > 2$.

   2 things worth noting. Firstly, the fundamental group is invariant under homeomorphism. Secondly, being path connected is invariant under homeomorphism.

4. (optional) Show that the Mobius strip $M$ does not have a retraction onto its boundary circle.