

Algebra 215: Problem Sheet 3

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1. Prove that if $H \leq G$ has index 2, then H is normal.
2. Show S_{n-2} is a subgroup of A_n .
3. Find all conjugacy classes in A_4 .
4. Show that A_4 has a normal subgroup of order 4. Use this to give a decomposition series for A_4 .
5. Let p be a prime. Show that a group of order p^2 is isomorphic to either $\mathbb{Z}/p^2\mathbb{Z}$ or $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$.
6. Let p be a prime, show that a group of order p^a has a subgroup of order p^b for every $b \leq a$. Hint: Use induction on a and that such groups have non trivial centre.
7. Let g_1, \dots, g_r be representatives from each conjugacy class in G assume that g_i and g_j commute for each i and j ($g_i \in C_G(g_j)$). Show that G is Abelian.